**ABSTRACT:**

The efficiencies of various types of water wheel are often quoted but it can be difficult to find a derivation. Only two types are discussed the overshot and the undershot. It is almost common sense to recognise that a greater efficiency will be realised when the wheel is overshot because we are using the weight of the water as well as the kinetic energy inherent in the water by virtue of its flow.

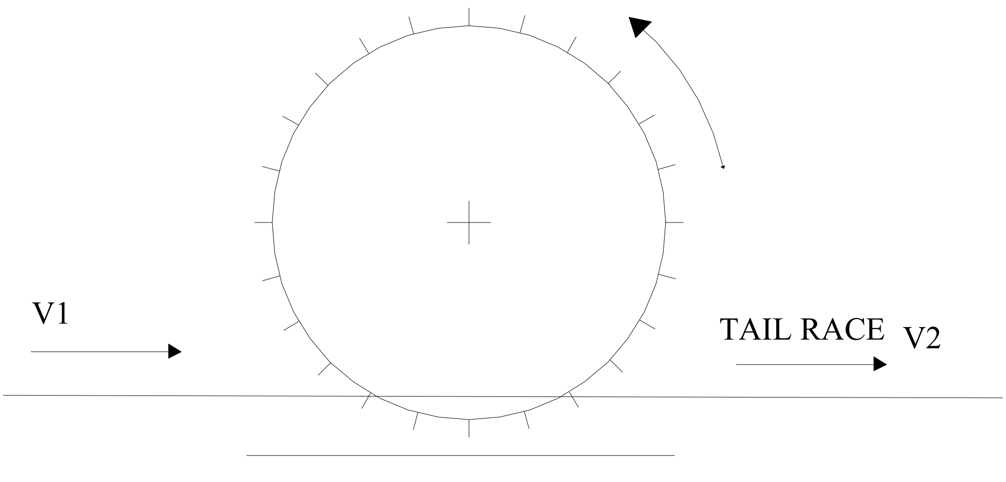
The author is by no means an expert on water wheels this document was simply the product of a question posed by a friend.

**INTRODUCTION:**

Water wheels have been with us some time and because of the spiralling cost of hydrocarbon fuels more people with the appropriate land and water are turning back to the water wheel for a cheap supply of power for heating and lighting.

This document looks albeit very superficially at water wheels and their efficiencies. There are a large number of variations of water wheels each with their own advantages and disadvantages. I have avoided the Pelton wheel which is far superior to a conventional water wheel one of the reasons for this is because it extracts nearly all of the available energy by virtue of the reversal of momentum due to the shape of its cups.

**UNDERSHOT WATER WHEEL:**




The volume of water hitting the blades is dependent upon the speed in the flume $V_1$; most treatments work out the volume then multiply by this speed to give a volume then a weight. Here I shall nominate W for the weight generated by the speed of the water flow, it is the same thing.

The change in momentum (m$\Delta$v) equals the mass times the change in velocity. The change in velocity $(V_1-V_2)$ is the final velocity minus the original velocity. Remember that the change in momentum is equal to the force.

The change of momentum $M_c$ is therefore given by:-

$$M_c = \frac{W(v_1 - v_2)}{g}$$

Why g? Because we want the mass not the weight. Therefore the pressure is given by

$$M_c = \frac{W(v_1 - v_2)}{g}.$$

We have to take into account that the blades are moving at $V_2 mS^{-1}$ Therefore the rate of work done on the wheel is given by:-

$$M_c = \frac{Wv_2(v_1 - v_2)}{g}$$

Now the kinetic energy of the water striking the wheel is given by:-

$$KE = \frac{WV^2}{2g} \text{ Joules}$$

The efficiency of the system is given by :- $\frac{\text{The energy given to the wheel}}{\text{The energy in the stream}}$

$$E = \frac{Wv_2 v(v_1 - v_2)}{g} \div \frac{Wv^2}{2g}$$

$$= \frac{2v_2(v_1 - v_2)}{v_1^2}$$

We can use calculus here for the maximum or minimum but it is a trivial exercise to see that the maximum will occur when $v_2 = 1/2 v_1$

$\therefore$

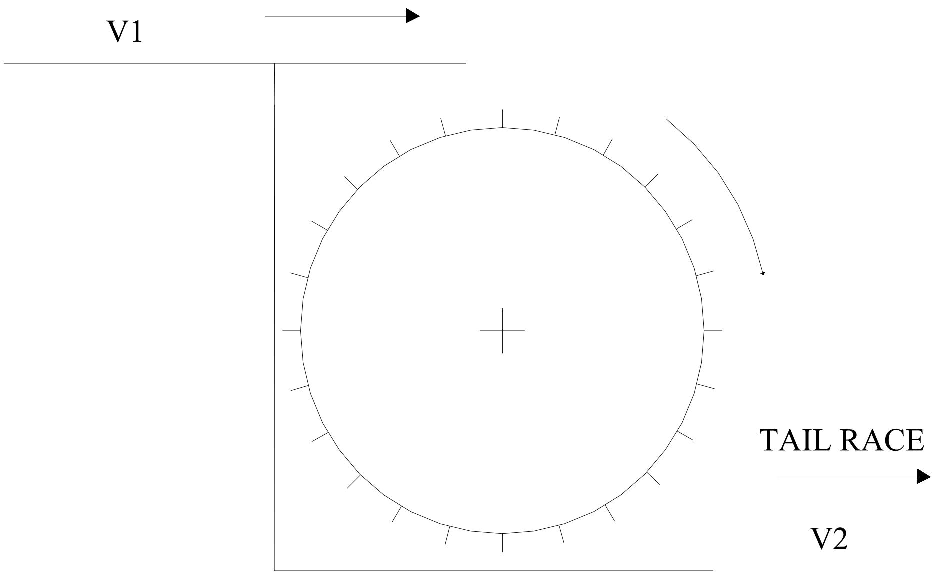
$$E = \frac{2v_2(v_1 - v_2)}{v^2}$$

$$= \frac{V_1(\frac{1}{2}v_1)}{v_1^2}$$

$\frac{1}{2}$ or 50%

**Thus we have shown that the maximum possible efficiency obtainable from an undershot wheel is 50% of the available energy.**

**OVERSHOT WHEEL:**




In the overshot wheel we must consider the kinetic energy of the flowing water as well as the potential energy provided by the height difference between the head race and the tail race. In essence we are applying the energy equation which is another way of expressing the conservation of energy.

That is the sum of the potential energy and the kinetic energy is equal to the sums of the rotational, translational and frictional losses. I will include the frictional losses in the overview but will not consider it in the analysis. This is a real and definite issue and when it comes to evaluating the efficiency of a real system must be included. We are therefore only looking at an idealised situation and not a practical one.

Once again the volume of water hitting the blades is dependent upon the speed in the flume $V_1$.
The change in momentum (m$\Delta$v) equals the mass times the change in velocity as with the undershot wheel. The change in velocity ($V_1$-$V_2$) is the final velocity minus the original velocity. Remember that the change in momentum is equal to the force. Remember too that we now had the added factor of the weight of the water in the buckets which assist in providing a turning moment.

The change of momentum $M_c$ is therefore given by:-

$$M_c = \frac{W(v_1 - v_2)}{g}$$

As before and

$$M_c = \frac{W(v_1 - v_2)}{g}.$$

We have to take into account that the blades are moving at $V_2mS^{-1}$ Therefore the rate of work done on the wheel by the moving water is given by:-

$$M_c = \frac{Wv_2(v_1 - v_2)}{g}$$

But we must now include the weight of the water and the height through which it drops.

$$M_c = \frac{Wv_2(v_1 - v_2)}{g} + V^2$$

Where did the $V^2$ come from? Mgh = $V^2$
This is a standard result which can be found in any physics text book

Now the kinetic energy of the water striking the wheel is given by:-

$$KE = \frac{WV^2}{2g} \text{ Joules}$$

As before the efficiency of the system is given by: - $\frac{\text{The energy given to the wheel}}{\text{The energy in the stream}}$

$$E = \frac{Wv_2v(v_1 - v_2)}{g} + v^2 \div \frac{Wv^2}{2g}$$

$$= \frac{2v_2(v_1 - v_2) + v^2}{v_1^2}$$

Now this time the efficiency is 1 or 100%.

**PRACTICAL CONSIDERATIONS:**

As can be seen below in the energy equation I have not included frictional losses; however Fitz Wheels quote a figure of merit of 90% for their product

Perhaps a better method of determining performance might lie with the energy equation quoted below:-

$$mgh + \tfrac{1}{2}mv^2 = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2 + F$$

Where

| | | |
|---|---|---|
| m | = | Mass. |
| V | = | velocity. |
| I | = | moment of inertia. |
| Ω | = | angular velocity. |
| F | = | Frictional losses. |

There are a number of considerations which will affect performance and design. For instance bucket design is very important. The buckets should retain their contents for as long as possible in order to maximise the energy transfer from water weight to wheel. The angle that they present to the flume carrying the water to the wheel is important. Reduction of splashing-anything which extracts energy from the water must be avoided. Buckets should also have a hole bored in the base of the bucket to prevent the trapping of air when the bucket is submerged. A great deal of energy can be lost to this.

Spacing and size of bucket is a very important design factor and care must be taken in evaluating the performance of the flume. If necessary the Darcy-Weisback formula for the calculation of lost head which is given by:-

Lost head (m) = friction factor (f) x length L (m) x Velocity head $\frac{V^2}{2g}$ (m)
Diameter D (m)

$$= f\,\frac{L \times V^2}{d \times 2 \times g}$$ can be used.

The determination of the moment of inertia (I) and the friction F of the wheel can be determined by the use of a weight say something like a house brick which has a weight of about 2.7 Kg offsetting the wheel by about 5° and allowing the wheel to turn noting the time when it slides off. We will nominate this as n this represents the number of turns made by the wheel made under the weight of the house brick and will obviously be a fraction because the brick will fall off before a complete revolution has been made.

Now the work done against friction in N revolutions is equal to the rotational kinetic energy as the wheel comes to rest.

$F = \frac{I\omega^2}{2N}$ if we now substitute this in the energy equation we now have:-

$$mgh + \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\,\frac{n}{N}$$

Now another way of expressing ω is v/r but the maximum tangential velocity of the water wheel is the same as the maximum velocity of the falling brick. Substituting for $\omega^2$

$$mgh = \frac{1}{2}I\,\frac{V}{r^2}^{2} + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\,\frac{n}{N}$$

Remember that the falling brick starts from rest and its maximum velocity can be found from the equation:-

$$h = \frac{(u+V)}{2}t$$

Which can be rewritten as:-

$$V = \frac{2h}{t}$$

More substitution will give

$$mgh = \frac{2Ih^2}{r^2t^2} + \frac{2mh^2}{t^2} + \frac{2Ih^2}{r_2t^2} \times \frac{n}{N}$$

$$\frac{2Ih^2}{r^2t^2}(1 + \frac{n}{N}) = mgh + \frac{2mh^2}{t^2}$$

$$I = \frac{(mght^2 - -2mh^2)r^2}{2h^2(1+\frac{n}{N})}$$

$$I = \frac{Nmr^2(\frac{gt^2}{2} - h)}{h(h(N+n)}$$

It should be borne in mind that the size of the wheel is related to the head of water by a set of rules which indicate that the optimum diameter is between three to six times the head of water and that power can be increased by making the wheel wider. This allows the wheel to carry more water.

Practical tests in the field also suggest that the most efficient energy transfer occurs when the wheel velocity is between 67-90% that of the water flowing in the delivery flume.

Another area of wasted power lies in the fact that the wheel is in contra rotation to that of the tail race. Methods have been devised to overcome this but these result in the loss of the kinetic element of the flow of the water.

Choice of bearing is important because journal bearings have a much higher frictional loss than ball bearings.

**CONCLUSION:**

These very simple derivations prove that an undershot water wheel is very inefficient and the best that can be achieved would be 50% of the available energy. In real terms that is more likely to yield 35-40%.

The overshot water wheel can theoretically yield 100% but as the analysis has shown this cannot be achieved due to unavoidable losses.

Efficiencies can, however, be achieved by careful design and planning**.**

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**USEFUL EQUATIONS:**

A comparison of useful rotational equations and linear equations is given here:-

| ROTATIONAL | LINEAR |
| --- | --- |
| $\omega^2{}_f = \omega^2{}_i + 2\alpha\theta$ | $v^2 = u^2 + 2as$ |
| $\theta = \omega^2{}_i + \frac{1}{2}\alpha t^2$ | $s = ut + \frac{1}{2}at^2$ |
| $\omega_f = \omega_i + \alpha t$ | $v = u + at$ |
| $\theta = \frac{(\omega_i + \omega_f)}{2}$ | $s = \frac{(u+v)}{2}$ |

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$$

If anyone would like any help you are welcome to contact me – other duties permitting billengineer@btinternet.com.